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Within each lesson there appears to be no grade-level separation. The advanced problems are mixed in with the easier ones. In Lesson One, for example, taking handfuls of squares to make up addition problems is included right along with finding series of consecutive whole numbers. How do we know which problems to do for which grade levels? .............................................................................................................. 157
I use something I call "Mad Minute" to time test my students on the basic facts. I find it very useful for drilling my students on quick recall. The students are given a minute to do the problems on each of several increasingly difficult pages. One page per minute, one minute per day. Once they can do one page at a mastery level, they pass on to the next sheet of problems for the next Mad Minute drill. My students love it and they learn so much from this test of speed. What harm is there in this? ................................................................................................................ 158

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In the books Mathematics Their Way and Mathematics a Way of Thinking, children made up words like "yuck" and "zurkle" to describe the groupings for the different bases. Why are made-up words not used in this book? ........................................................................................................................................331

Doesn't it confuse students to see 10 and not have it mean ten? Wouldn't it be easier to have our students say "ten" whenever they see 10 written, or "eleven" whenever they see 11, regardless of the base? Saying "one cup, one" seems so artificial. ........................................331

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