A K-6 Math Curriculum and One Big IF
Leaving No Child Behind - Holding No Child Back

The purpose of this chapter is to present the thinking that went into creating a K-6 math curriculum that truly leaves no child behind, while not slowing the advancement of any child who may learn at a faster rate.

While examples of curriculum are given, this chapter is not meant to be a curriculum guide. The curriculum guides are in the Center's *Mathematics Their Way* and *Mathematics a Way of Thinking* books and in the *Patterns and Connections* manuscript.

A college student in Turkey who was writing her PhD thesis on *Mathematics Their Way* asked me on which educational thinkers or educational philosophies Mary and I had based our teaching. I said Piaget was Mary's inspiration for the activities she created for *Workjobs.* The inspiration for my teaching was Keith Bush.

Keith Bush

I loved my Kindergarten class. It was a great first year in school. Spending days doing things like finger painting, playing dodge ball, taking naps, and making friends was really quite enjoyable. The only academic things we were asked to do that year were to learn to print our first names and to demonstrate that we could count to ten. Name-printing we did on our own. Counting to ten was something we were asked to do while standing up in class.

I could count to a hundred, but Mrs. Lewis had me stop at ten. When it was my friend Keith Bush's turn, he simply recited a few numbers in no particular order. I couldn't understand why he didn't know the order of the numbers. I also couldn't understand why Mrs. Lewis didn't ever teach him the patterns that I saw that made it possible for me to count all the way to a hundred.

Keith was a bottom student in Kindergarten and a bottom student every year after that. It was the teacher's job to teach and the student's job to learn. If a student failed to learn, it was the student, not the teacher, who received the failing grade. Keith was as destined to be a bottom student every year in school as I was destined to be at the top.

When I was in school, we were discouraged from helping our classmates with their work. The statement made was always, if you help him, he won't learn how to do it on his own. That's just how it was. It never occurred to me back then that there might be a different way.

I did wonder though why my teachers did not teach my classmates the tricks I saw that made learning math so easy for me. I later learned that what I saw as tricks were actually the patterns in math. And the reason my teachers did not share what I saw with my classmates was because they didn't see the patterns either.

I wish I had been Keith Bush's teacher back then, and not just his friend. Think how different Keith's life in school would have been if he had been taught in a class where every child learned and no child was ever left behind. As I began looking for ways to allow every child is my class to be a learner with no exceptions, my memories of my years in school with my friend Keith, who always struggled in school with math and every other subject as well, were and are always with me.

How Children Learn

Children are born ready to learn. A new born baby starts off knowing how to eat, sleep, smile, cry and poop. Everything else it learns it learns from scratch. What the baby has going for itself is its ability to think. Its little mind is a pattern seeking device, automatically setup to make sense out of its environment. Starting from scratch, the baby's mind will learn what words are, what they mean and how to say them. And learning language is just one small part of all the things the new born child will have learned before he or she enters school.

Let's imagine for a moment that responsibility for all this early learning that a child does is taken away from the child's parents. After all most parents are not credentialed educators. Why should we leave the child's early education the hands untrained amateurs? Instead, lets rely on textbook publishers to come up with an infant curriculum for baby-schools to use for teaching every child how to walk and talk. How well do we think that would go?

Fortunately, there is no need to take the child's early education away from the child's parents. Children are natural learners, and parents are rather good at learning, too. In addition, the whole of society is there to help. Help comes from the parents of the new parent's, the new parents' friends, the books on babies available, the pediatric advice from the baby's doctor, and the learning experience itself a new baby brings.

Making Learning Natural Again

Learning is a completely natural process until the child comes to school. Then the naturalness of learning is replace with books and workbooks that are the exact opposite of natural. To make it possible to teach every child while leaving no child behind, all we need to do is present what is to be learned in the same way the child has learned everything he or she learned before coming to school.

When a baby learns to talk, there is no formal curriculum required. All the parents needs to do is talk to their child constantly and then respond favorably to every almost-word the child learns to say back. To set up a natural environment for learning, we need to surround the child with the concept(s) to be learned. I'll use my friend Keith's situation as an example. Before Keith was asked to count to ten, there had been no counting activities at all taking place in our kindergarten class.

To surround Keith with the concept of counting to ten, counting could have been something everyone in class was already doing out loud everyday. For example, counting all the children in class each morning and writing the total number down to be saved for comparison with the totals from every other day. Are there more or less children today than yesterday? Why are the numbers the same (or different) today than yesterday? The numbers would go beyond ten, of course, but counting to ten would have been learned along the way.

There are hundreds of counting opportunities in every kindergarten class. And, the teacher can always say, "What shall we count today?" I said "doing out loud" for a reason. Keith could not count to ten. He was not the only one. His is simply the only failure that has stuck in my mind all these years. By having everyone count out loud, children like me who already know how to count to ten and beyond are modeling for our classmates who do not yet know, what the counting sequence is.

With each passing day, more children who did not know will become children who do, and there will come a time when everyone in class now knows, with no one left behind. Do you have any doubt that Keith would have been able to count to ten when asked if he had been allowed to learn to count naturally before he was asked that time?

Terrible at Math

It's not something a person boasts about if he or she is illiterate. However, there is no shame felt by people saying of themselves, "I am terrible at math!" In reality, however, if a person is really terrible at math that person cannot function in society. Math is an absolutely essential part of our everyday lives.

Parents do not let their children cross the street by themselves until that child had mastered the math involved. What does it take to cross the street safely? Looking both ways, of course. But looking for what? On-coming cars. Just seeing the car is not enough. The car's distance and speed must be assessed to know when it is safe to cross. Wake up in the morning and try to find some part of your day from that point forward that doesn't involve the use of math. Even the time we wake up is determined by a math calculation entered into our wake-up device.

I cannot think of very many things in our daily lives that do not use math. Cooking meals, getting anywhere on time, merging in traffic, meeting a friend for lunch, checking weather or traffic reports, taking a nap. Major events like planning a trip to Disneyland or Disney World, buying a house or a car. Math is everywhere all the time. We cannot and do not function without math.

Before my twin godsons entered kindergarten, I took them to a local beautician for their first professional haircuts. In the chit-chat that took place the beautician asked me what I did for a living. I mentioned that I gave mathematics workshops for elementary school teachers. She, of course, said “Oh! I’m terrible at math!”

The beautician was the owner of her business and had a handful of employees working for her. Being good at math was an essential part of running her business successfully. Keeping track of inventory, determining the amount of the fees to charge to meet her overhead costs and every other expense related to her business. Knowing how much time to allow for appointments, to have each one end in time for the next one to begin. Like everyone else who feels he or she is terrible at math, the beautician had confused being terrible at math in school with being terrible at math is real life. School math is the problem, not math itself.

Patterns

I said earlier that math was always easy for me because of the patterns that I saw. For me, then, the first and most important step in teaching every child would have to be the search for patterns in mathematics and everywhere else as well. In Marilyn Burn’s book *About Teaching Mathematics: A K-8 Resource*, Marilyn quotes Mary’s and my feelings about the importance of the search for patterns:

“Looking for patterns trains the mind to search out and discover the similarities that bind seemingly unrelated information together in a whole. A child who expects things to "make sense" look for the sense in things and from this develops understanding. A child who does not see patterns often does not expect things to make sense and sees all events as discrete, separate, and unrelated.”

Concept – Connecting - Symbolic

When we teach a child to talk, or at least allow a child to learn to talk, we do not write out a list of words we want the child to know. We simply surround the child with words. Eventually, we will show the child how these words are written using letter-symbols. However, we do not expect the child to understand what written words represent until that child understands what words are.

Textbooks and workbooks start with number-symbols and then present the rules for working with the symbols as things to memorize. Children who understand what the number-symbols represent, or are simply good at memorizing, do well. Children who do not yet grasp the concepts begin the process of being left behind.

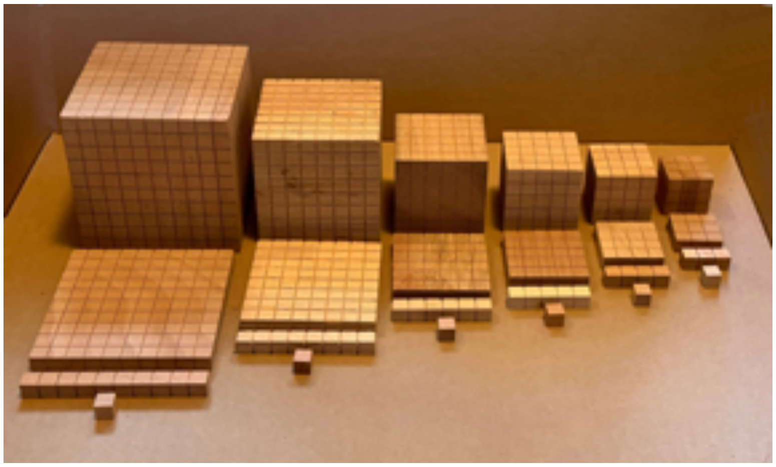
Mathematics is not rules to memorize, it is patterns to be seen. Numbers by themselves have no meaning. Numbers record concepts. To insure every child learns, we introduce the concept first and only when the concept is understood, do we connect the concept to its numbers. This is true for math. It will be shown later to be equally true for reading.

Thirty Teachers in the Class

Guiding philosophy: None of us is as smart as all of us. With 30 children in a class, each child has 30 teachers - the classroom teacher and the 29 other children, as well.

When I was a child in school, I caught on quickly and was, therefore, constantly separated academically from my friends like Keith. How could I, as a teacher, now expect to hold students like me back with students like Keith until Keith himself was ready to move on? Learning should not be a competition. It should be a shared experience. How then could I, as a teacher, make it so?

Dienes Blocks



In Chapter 8 - The Arithmetic Mistake and A Year Off From Teaching, I said that one of the many materials Bill Aho introduce me to in his math class was something called Dienes Blocks. The blocks were called Dienes Blocks because they were created by a Hungarian mathematician named Zoltan Dienes. The Blocks are multi-base, meaning they are concrete representations of different mathematical bases.

A set of Base 10 blocks has a small cube represent the number 1. The next size block is a narrow block 1 cube wide and 10 cubes in length, representing the number 10. Next is a flat square block, 1 cube thick, 10 cubes wide and 10 cubes long, representing the number 100. The fourth block is a cube, composed of 10 of the 100 blocks formed into a giant cube, 10 cubes high, 10 cubes long and 10 cubes wide, representing the number 1000.

The four different sized blocks represented the numbers 1, 10, 100 and 1000. I did not say they represented the numbers one, ten, one-hundred and one-thousand. These names are the names we have given to 1, 10, 100 and 1000 in Base Ten. As you can see, Base Ten is only one of the bases in the Dienes Multi-base Blocks. Bases 3, 4, 5, 6, and 8 are included, as well and all represent the numbers 1, 10, 100 and 1,000 in their respective bases.

The names we give 1, 10, 100 and 1000 in Base Ten are: one, ten, one-hundred and one-thousand. These names are for Base Ten only. For every other base, the numbers are said as one, one-zero, one-zero-zero, and one-zero-zero-zero.

Teaching Different Bases
My question for myself had been, "How can I make learning a shared experience?" When I saw the multi-base blocks, I had the answer. I would turn learning into a shared experience by starting with the assumption that no one knew anything. Since no one knew anything, all of my students would then start from the same "I don't know anything" starting point. Everyone together, no one ahead, no one behind, and we would keep it that way. Teaching different bases made this possible. Why was teaching in different bases the answer? Three reasons.

First
If I simply announced to all my students' parents that I will not move ahead with any lesson until every child understands, the reaction from the parents of the top students, and even from the top students themselves would be: That isn't right! It's so not fair!

However, teaching in different bases is not holding anybody back. I tell my class that students don't usually learn about different bases until they are at least in high school and sometimes not even then. All my students are impressed, including the top ones. Their parents are impressed, as well, especially since very few if any of them even know what different bases are, let alone how to solve problems using any number other than Base Ten. Their children will now be mastering math concepts that the parents themselves might find intimidating.

Different bases allow the children who catch on faster to never feel they are being held back by classmates whose learning needs more time. Students helping students doesn't keep the top students from being top students. It simply allows many more students to reach the top.

Second
When I introduce an activity in Base 4, not every student may understand the lesson that first day. When I introduce the same activity the next day using Base 5, the children who understood on day-one, will still understand on day-two, even when the base is changed. These students now become teachers for their classmates. With more teachers, more children will come to understand and, in turn, be available as teachers for any classmates who still need help with the next new base introduced on day-three.

Sometimes everyone will understand on day-one, but even for the more difficult concepts, I have rarely had it take longer that three-days and three different bases to bring everyone along. If it turns out a day-four is needed, there are still more bases to be used.

Third
Even if every child understands on day-one, we still repeat the lesson the next day using a different base, and then again on day-three using still another base. Each new day, we look for patterns in the activity. We will see patterns on day-one. Will we see that same pattern again on days two and three?

Searching other bases for the patterns they reveal and sharing with everyone what is seen, makes seeing the patterns we will find in Base Ten easier for everyone in class. Searching multiple bases for patterns also lays the foundation for the pattern searches we will be conducting continually in all areas of math.

Does It Work?
The Introduction - Credibility section of The Book of IFs showed the learning that resulted from my teaching different bases to my students. A class average of 3.5 on the pre-test in the Fall and a class average of 5.7 in the Spring. Every student experienced at least a year's growth in mathematics learning that year. In addition, as was said would be the case, the top students were still the top students, there were just more students at the top. And, if you define the bottom as any student who does not understand, there also was not a bottom any more, since every child had experienced more than a year's growth. If the "no child left behind" approach had been in use when all these children started school, the number of children who were below grade level in any year would be zero.

Not The Only Method Used
Arithmetic is not all there is to mathematics. I only used the teaching of different bases to allow my students who already understood basic arithmetic concepts to share their understandings with the students who did not. For every other concept in mathematics that I wished all my students to understand, different bases were not required. The search for patterns would be enough to allow understandings to be shared.

I will explain a few of the additional methods I used to insure that every child learned in a little while. But first, I will explain how the teaching of different bases became much more than just a method I developed for use in my fifth grade class.

Bob Uses That
In the 1970-1971 - Two Teachers of Teachers section of Chapter 9 - The Yearly History of a Change in Plans, I describe in detail how Mary and I became math instructors for the State of California Mathematics Specialized Teachers Project, also known as Miller Math. Mary was selected as an instructor based on her then soon-to-be published *Workjobs*. I was selected as an instructor because every time the person interviewing Mary showed her a material used by the program, Mary said "Bob uses that."

Mary and I were then teachers at Mayfair School in East San Jose. When we applied to teach at Mayfair, there were no openings for kindergarten or fifth grade teachers. A fifth grade opening was made for me by a teacher who switched to second grade. Since there were no kindergarten openings, Mary switched to teaching first grade.

Mary's curriculum was still based on the activities in *Workjobs*, with first grade activities now added in. *Workjobs* was the reason Mary had been referred to the Miller Math people as a potential instructor, but *Workjobs* itself did not include the use of a single one of the math materials on which she was now supposed to provide training to other teachers.

Mary could say "Bob uses that" because she and I were literally working side by side. I was near-by as she made each new Workjob and she was near-by as I acquired each new mathematics material for use in my own class. However, just as it never occurred to me to use any Workjob in my fifth grade class, it never occurred to Mary to use any of my math materials in either her kindergarten or first grade classrooms. Miller Math changed all of that. For Mary to be a Miller Math instructor, she had to change "Bob uses that" to "We both use that". The Mary and the Miller Math Experiences paragraphs below are taken from the Tribute to Mary included at the very beginning of the Center's *Mathematics Their Way Summary Newsletter.*

Mary and The Miller Math Experience

The closer our summer instructing assignment came, the more truly fearful Mary became. In the time since she had taken *Workjobs* to Addison-Wesley, we had changed schools and districts and Mary was now teaching first grade. At each successive grade switch, from fifth to second to kindergarten and then finally to first, Mary was concerned about how her new students would be to teach. Now she was going to teach adults. Adults were much too big! They knew too much! She would even be expected to teach fifth and sixth grade teachers, not just kindergarten and first grade teachers like herself.

Mary couldn't subtract, she couldn't multiply, she couldn't divide either, but that followed from the rest. She hadn't even used any of the materials she was now supposed to teach about. She'd seen my sets of Pattern Blocks, and my geoboards, and so on, but all she did in her class was *Workjobs*. *Workjobs* wasn't out as a book yet, so no one would know what she was talking about when she tried to explain what she did.

On top of all that, the leader of the instructional team with whom Mary would be working had let her know that she (the lead instructor) was not at all excited about having a member of the team who was, in effect, a trainee. It was made clear to Mary that she couldn't turn to the leader for help if she got in trouble. Mary simply had to pull her own weight.

With this as background, it's easy to see why Mary spent a part of each night of the first two-week workshop in tears. She knew she was awful in math. She knew she didn't have any idea what she could possibly teach the next day. But, she knew, too, that she really wanted to share with her fellow kindergarten and first grade teachers those activities she had created that made learning fun and exciting for children.

Mary often felt like quitting, because she felt so ignorant and out of place. But she also believed that what she had to share with teachers might make it so that their children didn't end up feeling about themselves as Mary had been made to feel about herself as a student. So, through her tears and anxiety, Mary doggedly worked at planning her lessons. Because Mary had no background in mathematics, planning each new lesson took hours. Before she could develop a lesson she had first to become a child again and learn with the materials herself.

My contribution was to sit with Mary each night and guide her through the learning experiences she needed to understand enough mathematics to share with the Miller Math participants the next day. My reward was watching Mary fall in love with mathematics and all its wonderful patterns as she passed, childlike, through all the learning experiences we shared together each evening.

I guided Mary through the experiences she needed to understand the secrets the materials had to share with her. However, I did not plan lessons for her. Once she understood what the materials offered, the same stream of inventiveness that had created *Workjobs* created an endless variety of child-like, child-centered activities to go with each new material encountered.

Mary had always needed the presence of some form of concrete representation in her learning. In her own schooling, real objects weren't a part of her environment. Now they were. This meant that Mary as a teacher had been allowed to find the key to her own learning that had eluded her as a student. All the activities she created reflected both her own thrill at discovering all there was to learn that she'd never learned before, and her personal conviction that children now should be allowed to learn in "Their Way".

We Both Use That

The end result of our first summer as Miller Math instructors was that, for the first time, Mary and I were now using the same math materials in our classrooms. Mary was now replacing the Workjobs she had developed as a kindergarten teacher with the new materials she had begun developing to share with her Miller Math participants.

I was sharing with the Miller Math participants activities I had used with my own students. Mary's situation was the reverse. She was now sharing with her first grade students activities she had created for use with her workshop participants.

My personal goal was to never leave a child behind and I was creating a curriculum that would let me meet my goal. The challenge at the fifth grade level was that many of the children who came to me had already been left behind, so not being left behind meant catching up with children who were already far ahead.

When Mary was teaching kindergarten, she didn't have to give much thought to children being left behind. As long as everybody learned, there was no catching up to do. And, all her kindergarten children had learned quite well.

Mary's first grade students were also doing quite nicely. However, *Workjobs* was a collection of separate activities, it was not an actual curriculum. It was like a great cookbook. Turn to any page and find something great to cook. However, there is a difference between following a recipe in a cookbook and taking a course on learning how to be a chef.

Workjobs aimed at making mathematics real for the child. Not something to memorize, but something to be understood. What was missing, though, was the connections to be made between all the separate activities.

I had formed the Miller Math materials I was using (and other materials, as well) into an inter-connected curriculum with the search for patterns as the connecting link. What Mary understood from the lessons I shared with her each night during our Miller Math experience was the importance of patterns and the search for them in making sense of math.

Beginning with her Miller Math experience, Mary began to learn the curriculum I had been creating for my fifth graders that incorporated all the "Bob uses that" materials. Learning my curriculum did not mean teaching my same lessons in her first grade class. It meant using the search for patterns to allow every child to fully understand mathematics. It also meant setting up a classroom structure that allowed every child in class to be every other child's teacher. Teaching every child. Leaving no child behind.

Teaching Different Bases in First Grade

My question for myself had been, "How can I turn my students' learning of mathematics into a shared experience?" To do this, I used different bases to make arithmetic concepts understandable for all my students, so the children who understood more quickly would be available for helping the ones who needed more time.

It did not occur to me that Mary would include my multi-base teaching strategy in her lessons for her first grade students. However, that is exactly what she did. I don't mean "exactly" as in copying me. I mean "exactly" as in using the same concept, but at a level appropriate for her first grade students.

Mary did not teach "Bases" she taught "Zurkles". Zurkles, or whatever other names the children made up for each new base. I emphasized for my students that they were learning different bases, to give credibility to the arithmetic lessons I was teaching. Mary simply used made-up names for each base and taught different bases without ever saying that was what she was doing.

Patterns Everywhere

Mary's creativity is quite evident in *Workjobs*. It is even more evident in *Mathematics Their Way*. Having learned the importance of patterns in making sense out of mathematics, Mary unleashed her creative talents to help her students see patterns everywhere.

The second chapter in *Mathematics Their Way* is entitled Pattern One. You do not need to read a single word in the text that accompanies that chapter's pictures to see the range of patterns to which Mary introduced her students before a single arithmetic concept is presented.

Examples of pattern search materials in the Pattern One chapter: Keys, shoes, dot strips, Unifix Cubes, the children in class, chairs, hands snapping and clapping, Pattern Blocks, Geoboards, Junk Boxes, necklaces, and more. Patterns once seen are then connected to patterns already seen or soon to be seen with another material.

A Curriculum

These are *Mathematics Their Way's* No textbooks – No workbooks – No worksheets Chapters

01 - Free Exploration
02 - Patterns One
03 - Sorting and Classifying
04 - Counting
05 - Graphing
06 - Number at the Concept Level
07 - Number at the Connecting Level
08 - Number at the Symbolic Level
09 - Patterns Two
10 - Place Value
12 - Pattern Book Experiments

Mary had gone from being terrible at math herself to creating a full-fledged early childhood mathematics curriculum.

In 1989, the National Council of Teachers of Mathematics (N.C.T.M) published its *Curriculum Evaluation Standards for School Mathematics*, commonly referred to as the N.C.T.M. Standards. One of the board members, touring the country that year to promote the adoption of the Standards by schools and districts everywhere, stated at each stop on his tour, "If you want to see the best example of how to implement the Standards in your primary classrooms, look to *Mathematics Their Way*."

If Mary Can Do It...

The general view of the teachers who knew Mary and me from our earliest days was the sentiment: If Bob can do it, that means Bob can do it. If Mary can do it, that means all the rest of us can do it, too. As I said earlier, I was always a top student. I was also a math minor, with a four-year post-college background that made me completely comfortable abandoning the current curriculum and starting over. My class was the only class our intern supervisors had filmed to show the other interns what teaching was supposed to look like. By itself, the fact that my fifth grade class did so well when compared to the other three classes at Mayfair School just meant that it was something I had done, and not necessarily what anybody else could do.

Mary was not a top student, at least not in math. Mary was a very a bottom student. As I said earlier, Mary could not subtract with any accuracy and she did not even know all her times tables. In addition to which, Mary had not taken any math classes since her Sophomore year in high school. The fact that Mary started off with so little understanding of mathematics turned out to be very helpful for her when she began teaching other teachers. Mary really understood what others teachers didn't know, because she had not known it either. Because Mary now knew it, she understood how to help other teachers come to know it, too.

Mary could introduce other teachers to all the ways she had now found to teach mathematic to her students, including the teaching of different bases, because she was Mary and if she could understood it and could do it, then they could do it, too.

The Other Methods Used

As I said earlier, I used the teaching of different bases to allow my students who already understood basic arithmetic concepts to share their understandings with the students who did not. For every other concept in mathematics that I wished all my students to understand, different bases were not required. The search for patterns would be enough to allow understandings to be shared.

What follows are examples of some of the additional methods and materials I used to replace the textbooks and workbooks I had rejected from the start. These are examples, not actual lessons. The actual lessons Mary and I used in our classrooms can be seen in the pdf copies of the Center's books available through the Center's website Home page.

Ceramic Tiles

A Multi-base Material
The Dienes Multi-base Blocks were my inspiration for teaching different bases to the students in my class. However, the blocks themselves were not the multi-base material that I used. My classroom lessons involved teaching every child at once. If I were to teach a lesson in Base 4, for example, I would need enough Base 4 materials for everyone in class. The Dienes Blocks did not meet that need.

One square inch ceramic tiles were the material that I used instead. The tiles are commonly available in sheets of 144. Fifty or so tiles for each student would require about 10 sheets for my whole class. Putting the sheets in a bucket of water separates the individual tiles from their backing. Sets of 50 or so tiles for individual student use can be housed in Tupperware containers or 10 X 10 individual pizza boxes.

The ceramic tiles were our counting units. We used the paper wrappers meant for cupcakes for our 10, 20, 30 and so on, holders for each base. We placed the tile-filled cupcake holders, which we call "cups", in paper bowls to hold our 100, 200, 300 and so on numbers.

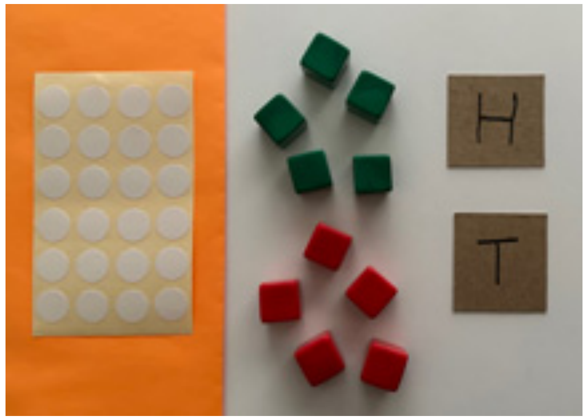
At the 00:53:22 mark of the first D-Talk video link on the Center's Home page, multiplication matrices in Base 3 through Base 10 are discussed. Ceramic tiles are much better suited for the exploration of multiplication patterns in different bases than are the Dienes Multi-base Blocks.

Other Activities As Well
The ceramic tiles are used for a wide variety of other activities, as well. One of many examples: I ask my students to multiply 25 x 25, 26 x 24, 27 x 23, 28 x 22, 29 x 21, 30 x 20, and so on. First question: Can you see the pattern for what problem I will ask you to multiply next? Second question: Look at the answer to each problem and see if you can predict what the answer to the next will be. My students have no difficulty with this activity since we share the answers to each question on their individual chalkboards.

I then have my students make a 10 by 10 rectangle with their tiles. Easily done, since they are already used to sharing their tiles with each other. The 10 by 10 rectangle take 100 tiles, which in my class means two students sharing. What do my students think I will ask them to do next? Change their 10 by 10 rectangle into an 11 by 9 rectangle. Any tiles left over? Now make a 12 by 8 rectangle. Same question. Continue the pattern for the problems and see what you can see.

In this simple example, students can see that the number pattern that they saw starting with 25 by 25 exists with tiles too, because all numbers do is record the patterns present in the materials they represent.

Dice and Cardboard Squares



The dice in the picture above have no numbers on them. The labels shown to the left are used to add the needed numbers to the dice.

No Worksheets and Yes Worksheets
Traditional dice are numbered from 1 to 6. My class does use traditionally numbered dice. However, the use of different bases is accompanied by a need for differently numbered dice, as well. For example, as my students are learning how to add and subtract numbers in Base 5, there will be a point when I will ask them to practice their addition or subtraction skills.

Traditionally, this would be a time when I would provide my students with a worksheet with addition or subtraction problems on it to be solved. The list of Ten No's includes No Worksheets for several reasons. The first is that worksheets separate students by ability. Students will complete any worksheets they are given at different rates. Some will finish a worksheet quickly, while others may not even have completed it by the end of the period.

A second reason is that the problems on the worksheets have predetermined answers that the teacher is responsible for providing. The students do the calculating and the teacher marks the calculations right or wrong. In my class, students create their own worksheets. Each base we work on has its own set of dice. For Base 5, each child is given two Base 5 dice. There are no 5's or 6's on Base 5 dice. The six sides of the dice are numbered 0, 1, 2, 3, and 4. The sixth side is given one of these five numbers.

Students use their Base 5 dice to create either addition or subtraction problems for themselves, depending on the arithmetic operation being practiced that day. The first two-dice roll is for the cups and tiles in the top row of the problem. The second two-dice roll is for the cups and tiles in the bottom row. Once the problem is created, students use their tiles and cups to solve it. The dice are used for subtraction problems as well, with a slightly different rule for their creation. The two dice are used to represent the cups and tiles, However, the top row always starts with a bowl included. 100 is the beginning. A two dice roll of 3 and 4 produces a top row of 134.

Students can use their tiles and cups to solve the problems However, in actuality by the time we begin the process of creating addition or subtraction problems to be solved, most, if not all, students have internalized the concept and connecting levels and are already operating at the symbolic level. They would already have experienced racing up and racing back activities. Using their Base 5 dice in groups of twos or threes, students race each other up to a bowl, or starting with a bowl, race each other back to zero. In my class whether the first person up or back or the last person up or back is the winner is decided in advance by the students themselves. In Mary's class, students use the more-less spinner Mary created to spin at the end to see if the first or the last person up or back won the race.

For my student-made worksheets, I provide my students with the dice and set the length of time they will be spending creating problems for themselves. The advantage of my saying, for example, I want you to create addition problems for yourselves for the next 20 minutes, is that no matter how many or how few problems each student creates and solves, all my students, fast and slow alike, will be finished at the same time. There will be no worksheet-caused separation by ability.

As my students are busy creating and solving their own problems, I circulate around the room looking at the problems they are creating. I tell them in advance what I will be looking for. In Base 5, I will be checking to see if everyone has remembered that in Base 5, there should not be any numbers 5 or above in any of their answers.

The Study of Probability
In the picture above there are two cardboard squares. Two are shown so that you can see one side is marked with an H for Heads, and the other side is marked with a T for Tails. While two are shown, each child only has one cardboard square to work with. The squares are a lead-in to the study of probability.

The first use my students make of their cardboard squares is for each student on his or her own to toss or flip his or her square and record the results on a sheet of graph paper. The head results are recorded in an H column and the tails results are recorded in a T column. The squares are tossed until either H or T reaches the top of its column. Once all of the students have had either heads or tails win, I ask for a show of hands for how many had heads win and how many had tails win and post the result for all to see.

Next, the students work in teams of two. If the class does not divide evenly into groups of two, one group of three is formed. Students now toss two cardboard squares and record the outcome of their tosses on a sheet of graph paper. Possible outcome for the tosses: H-H, H-T, T-T. Once again, the tosses are recorded until one of the three possibilities reaches the top of its column. And, once again, I ask for a show of hands for how many had heads-heads win or heads-tails or tails-tails.

For the heads-tails toss, there were students who had heads win and students who has tails win. This time, however, it is likely that all, or nearly all teams will report heads-tails as the clear winner. My question for my class is Why? Why did heads-tails win? If anyone can explain why, we can test his or her theory by tossing the two squares again and seeing if heads-tails still wins. If no one can think of an answer to my question, we toss the squares again to see if the results remain the same. I have asked the question. If my students do not know the answer yet, their next experience with the dice will give them a clue.

One Die and Then Two Dice.
For this activity we use standard 1 to 6 dice. Each student rolls his or her die and record the results on a sheet of graph paper. The results are recorded in columns for 1, 2, 3, 4, 5, and 6. The die is rolled until one of the six numbers reaches the top of its column. Once all of my students have had one of their numbers win, I ask for a show of hands for how many had each number win and post the result for all to see.

Next, the students work in teams of two. Once again, if the class does not divide evenly into groups of two, one group of three is formed. Student now roll two dice and record the outcome of their rolls on a sheet of graph paper. Possible outcome for the rolls, totals of 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12.

For the simple one die roll, it is likely that each one of the six numbers was a winner for some students. This time, however, the winning numbers are likely to be clustered between totals of 5 and 9 or perhaps 6 and 8. The totals of 2 and 12 will not be anybody's winners. My question for my class is once again Why? This time, though, I provide them a clue.

My students and I create a chart of the number of ways each number total can come up. For the number 2, there is one possibility: 1 and 1. For the number 3 there are two possibilities: 1 and 2, and 2 and 1.

Aren't 1 and 2 and 2 and 1 the same? No they are not, they are different. The reason I have two students roll two dice is to make clear the difference between 1 and 2, and 2 and 1. If Virginia and Brenda are rolling the dice, Virginia can roll a 1 while Brenda rolls a 2, and Brenda can roll a 1 while Virginia rolls a 2. That means there are two different ways that 1 an 2 can come up. We have different colored dice in my class. The same point can be made just as easily if one student rolls two dice, one green and one red. The number 3 can be made from red 1 and green 2, and also from green 1 and red 2.

The Chart of Possibilities

					4+3					
				3+3	3+4	4+4				
			3+2	4+2	5+2	5+3	5+4			
		2+2	2+3	2+4	2+5	3+5	4+5	5+5		
	2+1	3+1	4+1	5+1	6+1	6+2	6+3	6+4	6+5	
1+1	1+2	1+3	1+4	1+5	1+6	2+6	3+6	4+6	5+6	6+6
2	3	4	5	6	7	8	9	10	11	12

The chart of possibilities shows us the ways there are to make each number. The chart shows that the number 7 has the most ways to come up, 6 and 8 the second most, 5 and 9 the third most, and so on.

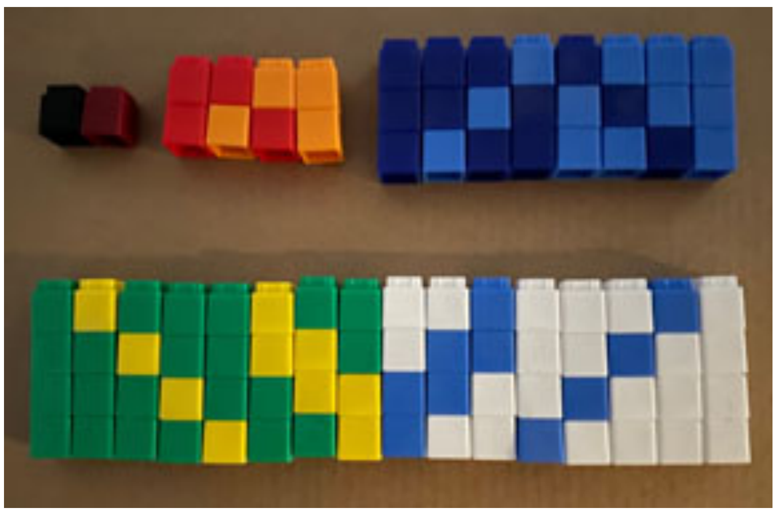
The same logic also applies to the tossing of the squares. H and T are twice as likely to come up as either H-H or T-T, because one person can toss heads while the second tosses tails, and then the roles reverse as the first person tosses tails while the second tosses heads.

Both Mary's and my students engage in many graphing activities. In Mary's class, the focus is primarily on gathering and observing the collected data. In my class we also begin looking for patterns in the data that may or may not allow us to anticipate the results to expect if we collect more of the same data.

As an example, if my class does a graph of the months of birth for everyone in class and the most common month is September, I ask, "Is our classes graph like a one die graph, apt to produce a different result for a different class? Or, is it like a two dice graph, showing us what we might expect from other classes, as well?"

The squares and dice provide the framework for my questions, and my students questions, as well. They are our lead-in to the study of probability. Our study of probability evolves over time into becoming statistical analysis (the science of collecting, exploring and presenting data to discover underlying patterns and trends).

Unifix Cubes and Patterns Everywhere
We use Unifix Cubes to look for patterns everywhere. Below is one of many examples. Ways to make 1 using two different colors. Ways to make 2 using two different colors. Ways to make 3 using two different colors. Ways to make 4 using two different colors. Two ways, then four ways, then eight ways, then sixteen ways. How many ways do you, the reader, think there might be to make 5 with two colors?



In Chapter 10, I referenced a demonstration lesson I gave to a class in Idar, India. The point I made then was that none of the student in the first two rows turned around to count their classmates when I asked, "How many children in the class?" Once we determined that there were 35 children in class, I then asked, "What fraction of the class are you?" Since no one in class volunteered an answer, I said that the answer to my question was 1/35 or one-thirty-fifth, and then I said why.

For fractions, one is whatever we say it is. If I say the class is ONE, then the bottom number of the fraction, known as the denominator, shows how many students are in that ONE. The top number of the fraction, known as the numerator, is the part of that ONE that each student is.

The lesson I presented in Idar was a fraction lesson. Along the top of the chalkboard at the front of the room, a portion of which you can see, I have written the numbers 1 through 36.

For my Idar lesson, the students would be exploring the fractions within each of the numbers from 1 through 36, with each number having its turn at being ONE.

I mentioned in Chapter 10 that the person doing the filming in Idar only filmed the first few minutes of my talk. When I stopped talking and the students started working, he stopped filming.

Right after I discussed with the students what fraction of the class each person was, I passed out twelve Unifix Cubes to each student.

I then had each student make a cube stick just two cubes long. That cube stick I said was now ONE. I then asked what fractions can you make with it? I demonstrated what I meant by breaking the two-cube stick into two separate cubes.

For a cube stick of two, the denominator would be two, because that's how many cubes were in the whole stick. And, the numerator for the individual cubes would be one, because each cube was just one cube. The fractions that could be made with a two-cube stick as ONE were one over two or one-half. I wrote 1/2 below the 2 on the chalkboard

I then had the students make a cube stick three cubes long. That cube stick I said was now ONE. I then asked again what fractions can you make with it? And again I demonstrated what I meant by breaking the three-cube stick into three separate cubes. The fractions that could be made with a three-cube stick as ONE were one over three or one-third. I wrote 1/3 below the 3 on the chalkboard

Next came the cube stick four cubes long. This stick was now ONE. For this cube stick I introduced a new rule that was not needed for the two and three cube sticks. You can break the cube stick more ways than just single cubes, as long as the pieces you break the cube stick into are of equal size. The four-cube stick breaks into four equal pieces. It also breaks in half with two cubes in each half. The fractions that could be made with a four-cube stick as ONE were one over four or one-fourth and one over two or one-half. I wrote 1/2 and 1/4 below the 4 on the chalkboard

The five-cube stick had only one way it could be broken into pieces of all equal size. So, only one over five or one-fifths could be made. And 1/5 was written below the 5 on h chalkboard. I used the cube sticks two through five to teach the students the rules for deciding which fractions could be made with each new cube stick. For the six-cube stick, I simply asked the class to see what fractions they could make with the six-cube stick as ONE and write their answers in their notebooks.

In my own classroom students write their findings on their chalkboards and then explain their answers to anyone whose answers are different than theirs. In Idar, I had the student share what they had written with the people around them. I then asked students to tell me what they had found and why.

Collectively, the class had no trouble deciding that the six-cube stick could represent one over six, one over three and one over two. I wrote 1/2, 1/3, and 1/6 on the chalkboard.

As the students worked on each new cube stick, I recorded what they found for each stick on the chalkboard. No student had enough cubes to make the larger cubs sticks, so they, of course, had to work together.

As I recorded their work, I asked them to look for patterns in what they had already done to see if they could predict what they would find for cube sticks not yet made. I asked if they could see patterns for which cube sticks make one-halfs, one-thirds, one-fourths, and so on.

On the following page is a record of what was on that chalkboard by the end of my lesson.

Cube Stick Fractions From 1 to 36

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{7}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{11}$	$\frac{1}{2}$	$\frac{1}{13}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{17}$	$\frac{1}{2}$
			$\frac{1}{4}$		$\frac{1}{3}$		$\frac{1}{4}$	$\frac{1}{9}$	$\frac{1}{5}$		$\frac{1}{3}$		$\frac{1}{7}$	$\frac{1}{5}$	$\frac{1}{4}$		$\frac{1}{3}$
					$\frac{1}{6}$		$\frac{1}{8}$		$\frac{1}{10}$		$\frac{1}{4}$		$\frac{1}{14}$	$\frac{1}{15}$	$\frac{1}{8}$		$\frac{1}{6}$
											$\frac{1}{6}$				$\frac{1}{16}$		$\frac{1}{9}$
											$\frac{1}{12}$						$\frac{1}{18}$

19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
$\frac{1}{19}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{23}$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{29}$	$\frac{1}{2}$	$\frac{1}{31}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{2}$
	$\frac{1}{4}$	$\frac{1}{7}$	$\frac{1}{11}$		$\frac{1}{3}$	$\frac{1}{25}$	$\frac{1}{13}$	$\frac{1}{9}$	$\frac{1}{4}$		$\frac{1}{3}$		$\frac{1}{4}$	$\frac{1}{11}$	$\frac{1}{17}$	$\frac{1}{7}$	$\frac{1}{3}$
	$\frac{1}{5}$	$\frac{1}{21}$	$\frac{1}{22}$		$\frac{1}{4}$		$\frac{1}{26}$	$\frac{1}{27}$	$\frac{1}{7}$		$\frac{1}{5}$		$\frac{1}{8}$	$\frac{1}{33}$	$\frac{1}{34}$	$\frac{1}{35}$	$\frac{1}{4}$
	$\frac{1}{10}$				$\frac{1}{6}$				$\frac{1}{14}$		$\frac{1}{6}$		$\frac{1}{16}$				$\frac{1}{6}$
	$\frac{1}{20}$				$\frac{1}{8}$				$\frac{1}{28}$		$\frac{1}{10}$		$\frac{1}{32}$				$\frac{1}{9}$
					$\frac{1}{12}$						$\frac{1}{15}$						$\frac{1}{12}$
					$\frac{1}{24}$						$\frac{1}{30}$						$\frac{1}{18}$
																	$\frac{1}{36}$

For the Idar class, this is all we did. For my own students, this is just a starting point. Next would be discovering rules for finding common denominators for adding fractions like 1/5 and 1/6.

Fraction lessons are just one of the many uses that can be found for Unifix Cubes in a classroom. Unifix Cubes demonstrate quite nicely how easy it is to teach every child basic math concepts with neither textbooks nor workbooks required.

Geoboards

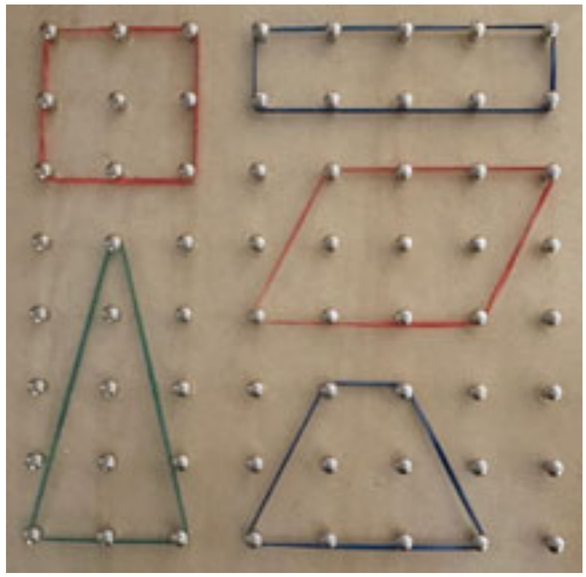
In The Book of IFs - Chapter 8 - The Arithmetic Mistake and a Year Off From Teaching I mentioned that Bill Aho introduced me to many very useful math materials I had not known of before and that I promptly began purchasing for use with my sixth grade students in the fall. Geoboards were the one material I was to use that I could make myself.

A geoboard is a piece of wood with evenly spaces nails on it. I went to the local lumber store, picked out an appropriate piece of wood and asked the person in charge of cutting to saw it into 30 equal pieces for me. He asked what the pieces would be for. I explained I was making a teaching material for my sixth grade class. He said his mother was a teacher, too. So he cut all the wood for me and waived the cutting fee.

Store-bought geoboards have the kinds of nails you can see in the image above. The nail's heads are rounded which serves to keep the rubber bands in place. For the geoboards I made for my class I used what are called "finishing nails". Finishing nails have heads that are no wider than the nails themselves.

To position the nails on the boards correctly, I made a cardboard template with small holes where the nails were to go. Once the template was taped in position on the piece of wood, I hammered a nail in each hole. Once all the nails were in place, the headless finishing nails made it possible for me to simply slide the template off and place it in position on the next piece of wood. Once the boards were completed, all I needed to add was some rubber bands

Formulas are Patterns Written Down



One of the uses of the geoboards is to look for patterns in the bases and heights of the five geometric shapes shown above to see if we can predict each shape's area.

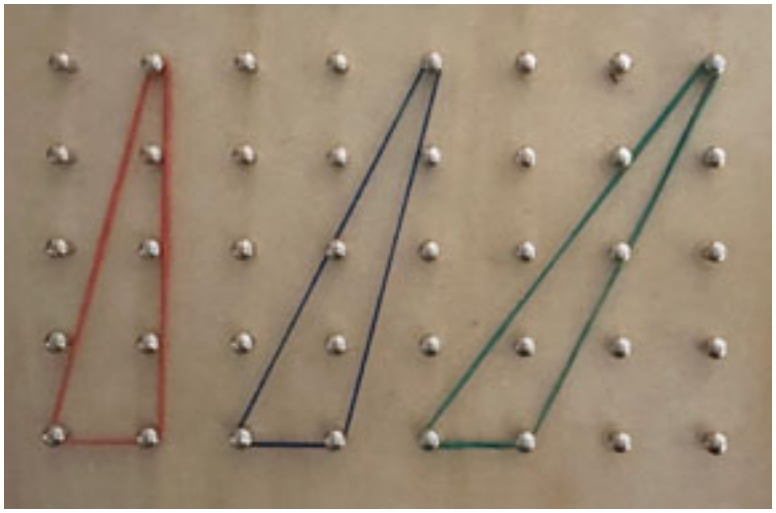
One shape at a time over several days or even weeks, I have my students record the number of spaces between the nails along the shape's base and its height and then calculate the shape's area. We do squares, then rectangles, then triangles, then parallelograms, then trapezoids.

My role as teacher is to teach my students how to find the area for each shape in its turn. Once my students can find the area for the shape we are working on, I have them record the bases, heights and areas for many different-sized shapes of that kind and use their data to see if they can see a pattern that will let them know what the area might be once they know the shape's base and height.

When a student or group of students thinks he or she or they see a pattern, I share that pattern with the whole class and ask everyone to see if that pattern works for all the areas they have found. If the pattern works for everyone, I show my students how mathematicians record that pattern in what mathematicians call a formula.

The formulas are as easily discoverable by my students as thy were for the mathematicians who first recorded them. Formulas are not things to be memorized. They are patterns to be written down.

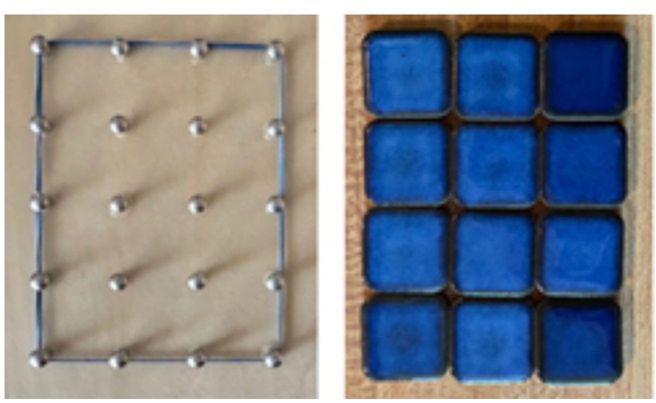
Height is Not the Same as Length



The three triangles above have the same base. What my students also learn is, they have the same height, as well. The rule for the five shapes for which my students look at bases and heights to predict areas is that the base for every shape must be along the lower edge of their geoboards. The height of a shape is not how long a side may be, it is how many spaces high it is.

The formula for the pattern my students find for triangles is one-half the base times the height equals the area. ($a = 1/2bh$). According to my students, all three of the triangles above have the same area. As the triangles are pulled to the right, does their area really stay the same? If you don't know, you can use a geoboard to find out.

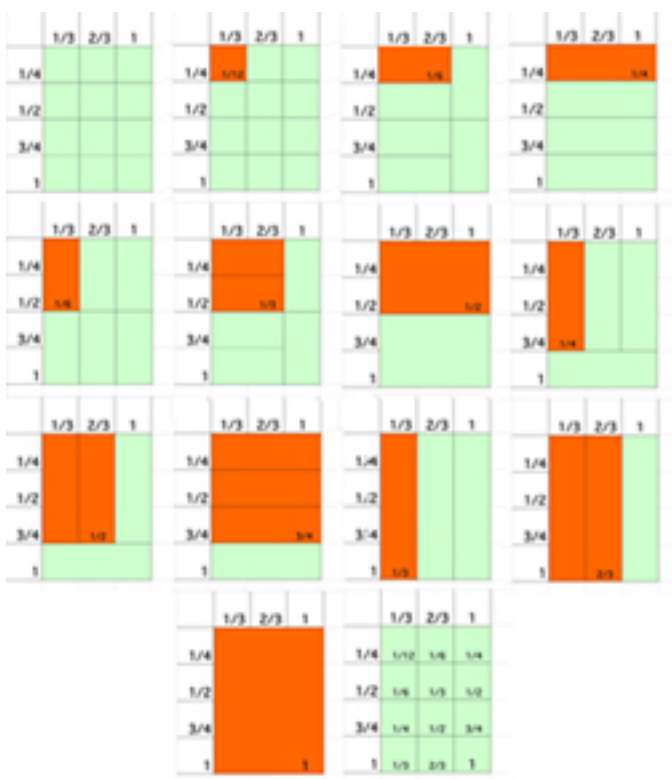
Fraction Patterns, Too



On the geoboard on the left, there is a rectangle that is 3 spaces wide and 4 spaces long. The area of the geoboard rectangle is 12 squares. The geoboard rectangle is equivalent to the tile rectangle to the right of it. Both rectangles represent the multiplication problem 3 x 4 = 12.

The rectangle represents something that is much easier for my students to see using their geoboards. My students learn in their Unifix Cube lessons on fractions, that one is whatever we say it is. For fraction lessons using geoboards, one is still whatever we say it is. The width of the geoboard rectangle is ONE width. The length of the rectangle is ONE length. The area of the rectangle is also ONE area. If the width of the rectangle is one then each of its spaces is one-third. If the length of the rectangle is one, then each of its spaces is one-fourth. And, if the area of the rectangle is one, then each of its squares is one-twelfth.

My students use their geoboards to make rectangles in many different sizes, all of which have in common that their lengths, widths and areas are all ONE. They record the various multiplication fractions problems these rectangles represent. They then look for patterns in their answers that will lead them to discovering the rules for multiplying fractions.



The image above shows the fractions my students find for the geoboard rectangle. The orange area represents the rubber band inside the rectangle. Students start by finding the fractions that are twelfths. They then see what other fractional value their rubber band might represent, using their Unifix break-apart rule that all the fractions must be the same size. Below on the left are the fractions found for a 12 Unifix cubestick. To the right of the cubestick list are the fractions to be found using a 3 by 4 geoboard rectangle.

12

$\frac{1}{2}$

$\frac{1}{3}$

$\frac{1}{4}$

$\frac{1}{6}$

$\frac{1}{12}$

1/4 × 1/3 = 1/12
1/4 × 2/3 = 2/12 = 1/6
1/4 × 1 = 1/4

1/2 × 1/3 = 1/6
1/2 × 2/3 = 2/6 = 1/3
1/2 × 1 = 1/2

3/4 × 1/3 = 3/12 = 1/4
3/4 × 2/3 = 6/12 = 1/2
3/4 × 1 = 3/4

1 × 1/3 = 1/3
1 × 2/3 = 2/3
1 × 1 = 1

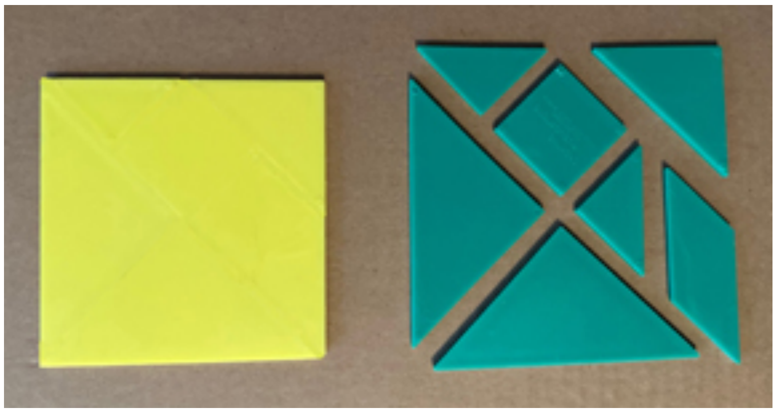
Students work together to see what multiplication of fractions problems their geoboards can be used to produce. To make sure everyone agrees to what has been found, the answer to the problems they find are shared with everyone else in class.

Once there are problems with answers, the students then look for patterns in their answers that would let them predict the answers to fraction multiplication problems for problems too big to workout on their geoboards.

The rules for multiplying fractions are not rules to be memorized, they are patterns to be to be discovered and understood.

Tangrams - Thinking Logically
For nearly all the math materials Mary was shown during her Miller Math instructor interview mentioned in the Bob Uses That section of Chapter 8 - The Arithmetic Mistake, Mary's said, "Bob uses that."

I do not know if the Tangram puzzle was one of the materials Mary was shown, but the Tangram puzzle was not something I had seen until it was shown to me and all the other Miller Math instructors at the training session we underwent before our first workshop that summer.



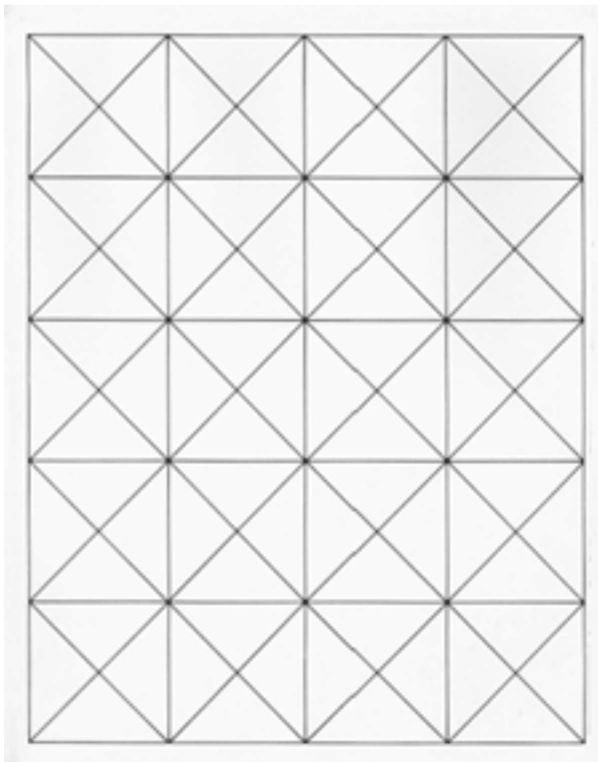
Tangrams are a Chinese puzzle that comes with a variety of stories about its creation. One such story: A sage was to take a precious sheet of glass to the king who needed a window in his palace. The square piece of glass was wrapped in silk and canvas and carried in the sage's backpack. Just before the sage arrived at the palace, he stumbled and fell down. The glass was broken. When the sage unwrapped the glass square, he was surprised to see that glass was not shattered but divided into seven geometric shapes. In his effort to recreate the original square, the sage moved the shapes around and made a variety of images. The king enjoyed the variety of geometric images and had the glass shapes recreated in wood. The Tangram puzzle had been created.

The most common use of the Tangram puzzle is to fit to its seven pieces onto already prepared outlines of shapes, moving the seven pieces around until the shape has been fully covered by the pieces. I personally really dislike being given tasks like that. The task starts with failure and continues to represent failure unless and until the shape to be match has been covered by the Tangram pieces.

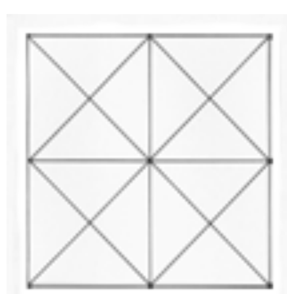
When I say "starts with failure" I am reflecting my own feeling when I am given something like that. I would not be given a Tangram shape to construct with the pieces unless it was a shape that could actually be made. That means to me that the whole time I have not yet fit the seven pieces on the image, I am failing.

I would not want to give anything like that to my own students. However, I came across a page in a book called *It's a Tangram World* that changed my thinking. It was not anything the book said. It was simply a page of triangles that the reader could use to make his or her own Tangram puzzle images.

This is the page of triangles I made that was inspired by the page I saw in the book:



The actual page is 8 1/2 by 11 and is now a blackline master in the *Mathematics a Way of Thinking* book. There are twenty small squares, with four triangles in each one. The small triangles on the page are the same size as the smallest two triangles in the Tangram puzzle. The square below with sixteen small triangles inside four small squares is the same size as the Tangram puzzle when its pieces are assembled in its original square shape.



That means the Tangram puzzle has an area of sixteen small triangles. The assignment for my students is to cut out shapes that are composed of exactly sixteen small triangles. Any shapes they wish to make is fine, as long as the sixteen triangles are connected in some way.

I collect all the cutouts and, once collected, I hand one cutout to each student. The assignment is either to fit all seven of the Tangram pieces on the cutout or to explain why you don't think it can be done by you or anyone else.

If a student has successfully placed his or her pieces on the cutout, he or she shows it to a classmate, they both put their names on it, and the puzzle is pinned in the "Yes" area of the space I have set aside on the classroom's bulletin board.

If a student thinks no one could put all their pieces on a particular puzzle, that student explains his or her thinking to a classmate and they both come to me with their explanation. If they convince me, then they both put their names on the shape along with mine and pin it to the "No" section of the bulletin board.

Once one shape is determined to be either a yes or a no, the student selects another shape from the box that houses them and the process begins again. When we run out of shapes, more are easily made.

Some shapes are easily found to be a "Yes", like the square shape. Some shapes are found to be easily "No", like any shape that has no place to fit the larger triangles. However, most shapes fall somewhere in the middle and the only way to know if they are either a Yes or a No is to try fitting all seven shapes on them.

I said what I did not like about the Tangram puzzles was that the puzzles start with failure and continue to represent failure unless and until the shape to be match has actually been reproduced with the Tangram pieces. That was because the only puzzles we were given to solve were ones that had solutions.

Now the activity does not start with failure. When a student cannot match all his or her pieces to the shape, it is possible that the shape is one that cannot be done by anyone. Rather than just move the pieces around until the predetermined answer has been found, the student now has to think about what is and what isn't possible. What is possible is demonstrated by fitting the seven pieces on the shape. Proving that a shape is one that no one could ever fit the pieces on requires a different kind of thinking.

In *Mathematics a Way of Thinking*, the chapter on Tangrams is called "Tangrams Logical Thinking". Tangrams are, for my students, an exercise in logical thinking.

My Eight Questions

Mathematics at all levels is simple and basic and straight forward. So, too, are the questions that we teachers may ask to encourage student explorations. There is no limit to the questions we may ask our students, because there is no limit to the possibilities they show us as we observe them at play and at work. The more days or weeks or years we spend watching our students, the more questions they will have taught us to ask.

Below are eight basic questions or statements that I use to convert my observations into questions that encourage my students' explorations:

1. What can we find out about (or make with) (or do with) this material?

2. What would happen if...?

What would happen if we put this rock in the water container?

3. If you can make it (or do it) with... can you make it (or do it) with...?

You have made an A-B-B pattern with Unifix Cubes. Can you make an A-B-B pattern with buttons? Or people? Or... ???

4. Can you do it a different way?

You have made a very nice bar graph to show the favorite television shows for the children in our class. Now, can you think of how to make your graph a different way?

5. How many ways can you...?

How many ways can you divide these twelve squares into groups of equal size? How many ways can you make shapes with an area of two square units on your geoboard?

6. Do you see a pattern? Have you seen it before?

Or, rephrased at a more advanced level: Organize your data and look for a pattern.

Look in the first column of your place-value recording strip. Do you see a pattern for the numbers there? Have you seen that pattern before? Where have you seen it? Is there a pattern in the cups column as well?

We asked everybody in our class to tell us in what month they were born. Lets put what everybody in our class told us in a graph, so we can see if one month is more common than another. Would our most common month be the same month for the other classrooms in our school? Would a graph for when our parent were born look the same as our graph? Is there a pattern that would let us know the month when the most people are born?

7. Predict what will happen if...
Or, rephrased at a more advanced level: Use the pattern you see to help you know what will happen next.

We found patterns on our recording strips for Plus One in Base 4 and Plus One in Base 5. Can use these patterns to help us know what will happen when we make a recording strip for Plus One in Base 6?

8. Find the one (or ones) that does not (do not) work.
Brenda's group says that for their geoboard triangles, the pattern they have found for predicting the area is to take the base times the height and then divide that number in half. This seems to work for their triangles. Have the people in your group find if there are any triangles your group has found for which this pattern does not work.

The eight questions and statements listed above are meant to give us a starting point. They are not meant to stifle our own creativity and inventiveness. The questions are meant to be mixed together and matched with our own thoughts. They are not meant to be a checklist of what we must say. The more comfortable we become in asking, the more we can think of what to ask.

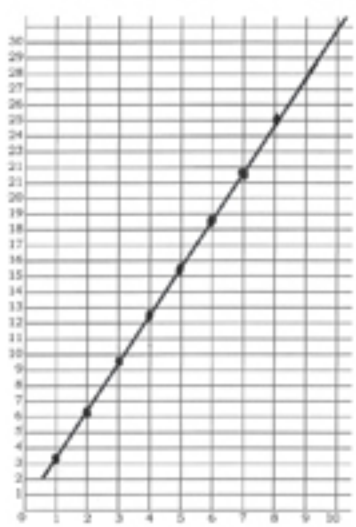
Saudi Arabia
I mentioned in Chapter 10 - The Ten No's and Starting from Scratch, that there was only one other time beside my Idar, India experience when I have been asked to teach any children in conjunction with my teacher-training visit to a school or district. That earlier time was April, 1992 in Jeddah, Saudi Arabia.

Over the course of my eight-day stay in Jeddah, I was asked to give demonstration lessons in every grade from first to fifth or sixth while the teachers at that grade level looked on. I don't remember if the school included sixth grade or stopped at fifth. I do remember, though, I quite enjoyed the experience.

On the Friday I was there, several of the teachers from the school and a few of their spouses and I had a very nice picnic in the dessert. One of the spouses happened to be an eighth grade math teacher. He told me that his wife really enjoyed the math lessons I had been presenting, but that the use of manipulative materials to present concepts would not work for older students like his.

The desert offered quite a bit of sand for my use, so I drew a very large L on it. I said the bottom of this L will be for the diameter and the side will be for the circumference. The picnic had many circular objects, including plates and lids of various sizes. I then had Mr. Spouse take each plate and lid separately, mark its diameter along the bottom of the L and then roll out and mark its circumference on the side of the L. For each set of diameters and circumferences I had him draw a line strait up from the mark for the diameter and strait across from the mark for the circumference and place a pebble in the sand where the lines intersected.

This was, of course, before we all had mobile phones that gave us cameras wherever we were, so I do not have a picture of what that row of pebbles looked liked. However, it looked something like this:



Our sand graph did not have numbers along either its X or Y axis, but it was, of course, obvious that all the pebbles were in a straight line.

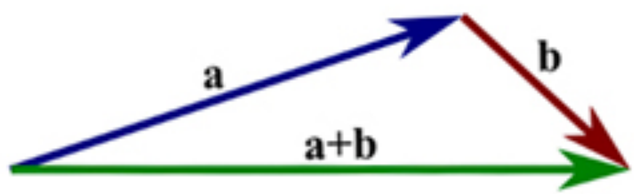
I told Mr. Spouse that if we actually had numbers for the diameters and circumferences, he would be able to see that the slope of that line of pebbles was a number just a little bit bigger than three. In fact, it would be 3.14959 with a lot more numbers added on. The name that mathematicians give the number for the slope of that line is Pi. I then asked him if he had ever let his students do the same kind of graph of diameters and circumferences so they could discover Pi for themselves. He said he himself had never even done it, but he was definitely going to do it with his class now.

Students at every level deserve the chance to substitute memorizing what the mathematicians who have come before them have discovered and be allowed to discover it for themselves.

Learning for Passing Tests In School
At a mathematics conference where the Center had a booth, a group of high school students were there to promote a math program from another booth. The students were boasting to everyone of the greatness of that company's Algebra textbooks and workbooks. When they came to the Center's booth, I asked them, "What do you use Algebra for?" Their answer, "We use it for passing tests at school."

I took Geometry as a Sophomore in high school. In addition to the standard Euclidian Geometry, we learned something called Vector Math. Vector Math made no sense to me. However, my goal was Stanford and if learning nonsense was what it took (and it often did), then learning nonsense was fine with me.

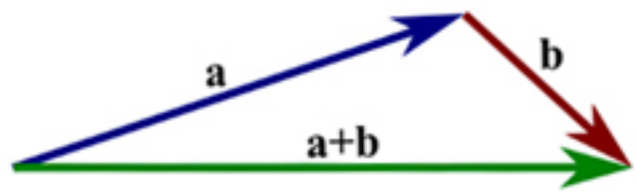
Vector math involves learning how to add arrows or vectors together. Vector-a added to Vector-b equals Vector a+b. That made no sense to me. How can the blue line plus the red line be said to equal the green line? But that's what Vector Math said the answer was.



I could not understand why adding arrows was included in a geometry class. I also could not understand why anybody would want to add arrows in the first place. But, of course, I learned Vector Math for passing tests in school, because learning nonsense was fine with me.

The Toughest Class
When I was at Naval Officer Candidate School (OCS), the class that gave the most students the most trouble was Operations. And, the lessons in Operations that gave students the most difficulty had to do with relative motion. Your ship at sea traveling at night spots the light of another ship in the distance. If your ship were sitting still, you could track that dot and accurately determine its speed and direction. However, your ship is moving, so what that dot shows you is not its actual speed and direction. What you see is it its relative speed and direction compared to your own ship's speed and direction.

There is a way to calculate the other ship's actual speed and direction, once you determine its relative speed and direction. In the Operations class the method was taught as operations on a maneuvering board. I knew it by a different name, "Vector Math". In navigation the vectors representing each ship's course and speed are plotted, to determine if the ships are likely to collide.



Vector arrows measure two things at once – in this case, speed and direction. Vector-a is your ship's actual speed and direction. Vector-b, which is placed at the end of Vector-a, is the other ship's relative speed and direction. Vector a+b is the line drawn from the start of Vector-a to the end of Vector-b. Vector a+b is the actual speed and direction of that second ship. Vector Math is, in the real world, a useful tool.

Operations was such a tough class that the student in my section who was a Yale Law School graduate was in danger of failing it. Rolling-out was the penalty for failing any class, no matter how well you were doing in all the others. In week sixteen, so many students did poorly on the Operations final exam that OCS had to add what was called a "J-Factor". A J-Factor was an artificial number added to everyone's test score, to reduce the number of failing grades to an acceptable level. My law school graduate friend was one of the students who owed his OCS graduation to that J-Factor added to his Operations final grade.

Making Mathematics Meaningful

The response of, "We use it for passing tests at school" given by the high school students at that math conference is not surprising. I learned Vector Math in high school because it was required of me. I saw no purpose in its use but it made no difference to me. It would be on the test, so I had to know it. Throughout my many years in school I frequently learned what I regarded as nonsense.

Now, as a teacher I will not teach my students nonsense. It is my responsibility to make mathematics meaningful. My geometry teacher in high school may have known that Vector Math was actually useful, but if he did, he never shared that knowledge with any of his students. If I cannot explain to my students the used they can make of what they are learning, then why am I teaching it?

Leaving No Child Behind - Holding No Child Back

Not all students are equally good at memorizing. However, all students are capable of understanding.

In the Credibility Introduction to The Book of IFs the pre- and post-test results of the math test administered to the four fifth grade classes at my school were presented. If you have read that chapter, you know my entire class, fast and slow, did quite well on that exam. The faster students were never separated out. They spent that entire year sharing their understandings in math, and in every other subject as well, with every other student in my class.

Explaining concepts to another child helps the child doing the explaining understand it even better than if that child simply kept his or her understandings to him or herself. When learning becomes a shared search for patterns, with everyone sharing equally what is found, every child is involved in every other child's learning and every child learns with no child ever left behind and no child ever held back.

A K-6 Math Curriculum's One Big IF

If Mary had not taken my ideas for teaching mathematics to my fifth grade students and converted them for use in the primary grades, they never would have been known outside of my own classroom. Meaningful change in curriculum does not begin in fifth grade, it begins in first. It never would have occurred to me to introduce kindergarten and first grade students to Bases other than Ten and their patterns to be seen.

Miller Math was the reason Mary learned my approach to teaching mathematics to every child in class without leaving any child behind. However, Miller Math did not make Mary who she was. It was Mary's own creativity that she brought to bear in creating an innovative math curriculum for the primary grades that allowed every single child to learn while never leaving any child behind.